

## Math 304 (Spring 2015) - Homework 1

Solve the following linear systems:

**Problem 1.**

$$\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 = 5 \\ 2x_2 + x_3 - 2x_4 + x_5 = 1 \\ 4x_3 + x_4 - 2x_5 = 1 \\ x_4 - 3x_5 = 0 \\ 2x_5 = 2 \end{cases}$$

**Solution:** The system is in strictly triangular form. We can use back-substitution.

$$\begin{cases} x_5 = 1 \\ x_4 = 3 \\ x_3 = 0 \\ x_2 = 3 \\ x_1 = -2 \end{cases}$$

**Problem 2.**

$$\begin{cases} -x_1 + 2x_2 - 3x_3 + x_4 = 1 \\ -x_1 - x_2 + 4x_3 - x_4 = 6 \\ -2x_1 - 4x_2 + 7x_3 - x_4 = 1 \end{cases}$$

**Solution:** Write down the augmented matrix:

$$\left[ \begin{array}{cccc|c} -1 & 2 & -3 & 1 & 1 \\ -1 & -1 & 4 & -1 & 6 \\ -2 & -4 & 7 & -1 & 1 \end{array} \right]$$

use elementary row operations to find its row echelon form

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & -6/17 & -2/17 \\ 0 & 1 & 0 & -5/17 & 72/17 \\ 0 & 0 & 1 & -7/17 & 43/17 \end{array} \right]$$

$x_4$  is a free variable, we have

$$\begin{cases} x_1 = \frac{6\alpha-2}{17} \\ x_2 = \frac{72+5\alpha}{17} \\ x_3 = \frac{43+7\alpha}{17} \\ x_4 = \alpha \end{cases}$$

**Problem 3.**

$$\begin{cases} x_1 + 2x_2 - x_3 = 1 \\ 2x_1 - x_2 + x_3 = 3 \\ -x_1 + 2x_2 + 3x_3 = 7 \end{cases}$$

**Solution:** Write down the augmented matrix:

$$\left[ \begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 2 & -1 & 1 & 3 \\ -1 & 2 & 3 & 7 \end{array} \right]$$

use elementary row operations to find its row echelon form

$$\left[ \begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 1/2 & 2 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

use back-substitution we have 
$$\begin{cases} x_1 = 1 \\ x_2 = 1 \\ x_3 = 2 \end{cases}$$

**Problem 4.**

$$\begin{cases} x_1 - 3x_2 + x_3 = 1 \\ 2x_1 + x_2 - x_3 = 2 \\ x_1 + 4x_2 - 2x_3 = 1 \\ 5x_1 - 8x_2 + 2x_3 = 5 \end{cases}$$

**Solution:** Write down the augmented matrix:

$$\left[ \begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 2 & 1 & -1 & 2 \\ 1 & 4 & -2 & 1 \\ 5 & -8 & 2 & 5 \end{array} \right]$$

use elementary row operations to find its row echelon form

$$\left[ \begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 1 & -3/7 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$x_3$  is a free variable, we have 
$$\begin{cases} x_1 = \frac{2\alpha+7}{7} \\ x_2 = \frac{3\alpha}{7} \\ x_3 = \alpha \end{cases}$$

**Problem 5.**

$$\begin{cases} x_1 - 2x_2 + 4x_3 = 2 \\ 2x_1 - 3x_2 + 5x_3 = 3 \\ 3x_1 - 4x_2 + 6x_3 = 7 \end{cases}$$

**Solution:** Write down the augmented matrix:

$$\left[ \begin{array}{ccc|c} 1 & -2 & 4 & 2 \\ 2 & -3 & 5 & 3 \\ 3 & -4 & 6 & 7 \end{array} \right]$$

use elementary row operations to find its row echelon form

$$\left[ \begin{array}{ccc|c} 1 & -2 & 4 & 2 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 3 \end{array} \right]$$

This system is inconsistent.